Enrollment No: _____ Exam Seat No: _____

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Graph Theory

Subject Code: 5SC04GRT1 **Branch: M.Sc. (Mathematics)**

Semester: 4 **Date: 26/04/2018** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Answer the Following questions:** **(07)**

a) Draw a simple graph with 5 vertices and 8 edges. **(02)**

b) Define: Arborescence **(02)**

c) Define Strongly connected diagraph and draw it. **(02)**

d) Find out from the following statement which is correct. **(01)**

i) Every cycle is closed walk.

ii) Every closed walk is cycle.

Q-2 **Attempt all questions** **(14)**

a) Let G be a simple graph with n vertices and k components then the graph G have at most $\frac{(n-k)(n-k+1)}{2}$ edges. **(07)**

b) Define minimal connected graph and prove that if G is a minimal connected graph if and only if it is a tree. **(05)**

c) Define: 1) Distance between two vectors 2) Clique **(02)**

OR

Q-2 **Attempt all questions** **(14)**

a) State and prove necessary and sufficient condition for disconnected graph. **(07)**

b) Answer the following questions from the following graph **(05)**

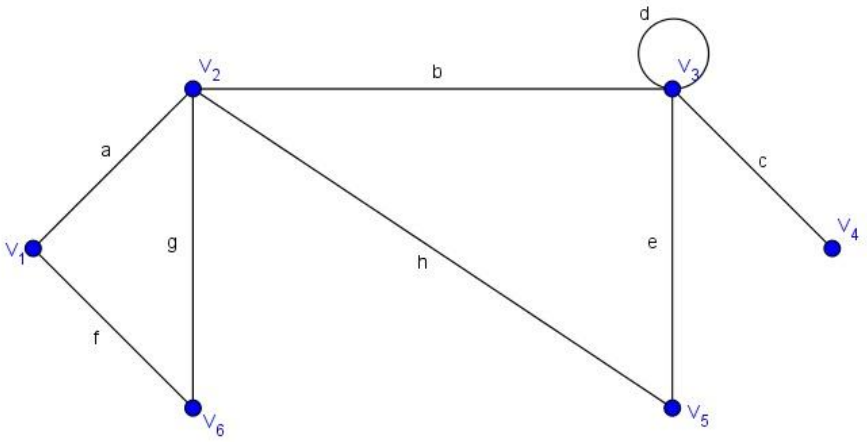

Figure – 1

i) Write one Spanning tree.
ii) Write one fundamental circuit w.r.t. i).
iii) Write adjacency matrix.
iv) Write one closed walk of length 7.

c) Verify first theorem of graph theory for above graph. **(02)**

Q-3 Attempt all questions **(14)**

a) A diagraph *G* is an Euler diagraph if and only if it is connected and balanced. **(05)**

b) From the following adjacency matrix draw the diagraph *G*. Also find X^4 and hence find the directed edge sequence of length four from v_2 to v_3. **(05)**

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

c) Define the following: **(04)**
1) Asymmetric diagraph 2) Complete symmetric diagraph
3) Fragment 4) Node base

OR

Q-3 Attempt all questions **(14)**

a) Draw a diagraph and construct longest circular sequence of 1's and 0's such that no subsequence of 4 bits appears more than once in the sequence. **(05)**

b) Let *G* be a connected diagraph with n vertices then the rank of $A(G)$ is $n-1$. **(05)**

c) Let *G* be an out-tree then *G* is tree in which every vertex than the root has exactly one out degree. **(04)**

SECTION – II

Q-4 Answer the Following questions: **(07)**

a) Define: Dominating set **(02)**

b) Define: Planner graph **(02)**

c) Define: Chromatic polynomial **(02)**

d) If $E \neq \phi$ in any graph G then chromatic number of G is _____. **(01)**

Q-5 Attempt all questions **(14)**

a) Prove that the vertices of every planner graph can be properly colored with 5 colors. **(07)**

b) Find chromatic polynomial of following graph. **(07)**

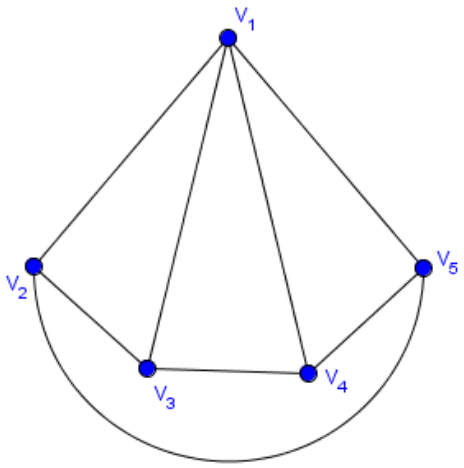

Figure – 2

OR

Q-5 Attempt all questions **(14)**

a) State and prove Dirac's theorem. **(09)**

b) Show that the following graphs are isomorphic. **(05)**

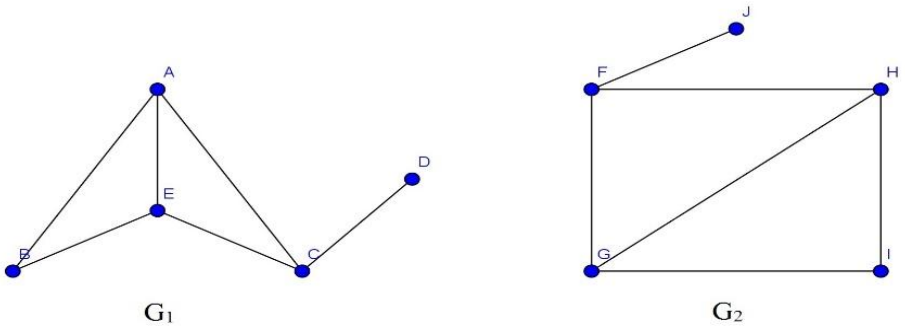

Figure – 3

Q-6 Attempt all questions **(14)**

a) State and prove Hall's theorem. **(10)**

b) Answer the following questions from the following graph **(04)**

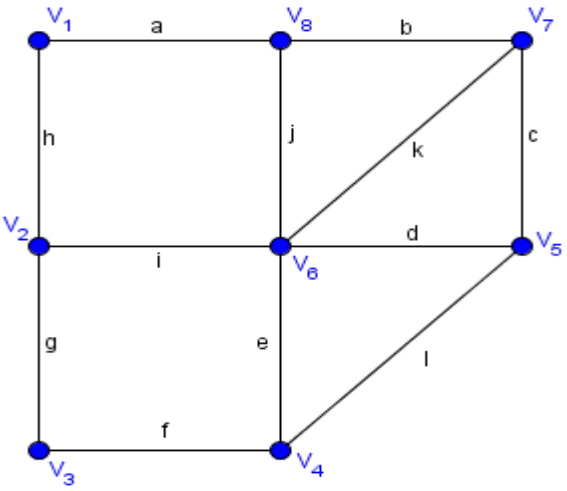

Figure – 4

i) Find a perfect matching and a maximum matching.

ii) Find one M-augmenting path and M-alternating path.

OR

Q-6 Attempt all Questions **(14)**

a) State and prove Min-Max theorem. **(10)**

b) Answer the following questions from the following graph **(04)**

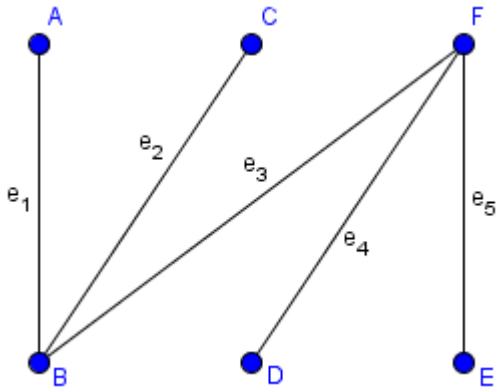

Figure – 5

i) Find a vertex cover and minimum size of vertex cover.

ii) Find a edge cover and minimum size of edge cover.